DETERMINATION OF THE EXTERNAL TEMPERATURE – FORCE FIELDS FOR EQUAL – STRENGTH STRUCTURE ELEMENTS DURING CREEP

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Acquaintance with the analysis and design of equal-strength structure elements operating under creep conditions can be made in [1-4], e.g., where the body geometry (structure element) is determined for given loads and temperature such that it would be equally strong. In all the computations noted above, the process of damageability of the material is not taken into account, and only the steady-state creep equations are used here. The body is considered heated uniformly. In certain cases, equal strength of a body with a previously assigned geometry can be realized during creep by a suitable selection of the external load and temperature fields. This paper is devoted to this question. The body is considered nonuniformly heated here. A system of equations describing all three stages of material creep with the cumulative damage process taken simultaneously into account is used. The method of determining the external loads and temperature is given for the axisymmetric plane strain case. The temperature field is here considered planar and axisymmetric.

A body (structure element) heated nonuniformly and loaded by external forces will be called optimal in longevity (or equally strong during creep) if damageability at all its points proceeds in an identical manner, and therefore, simultaneously after a previously assigned time t_{**} the damageability parameter ω reaches its critical value of one. It is shown in [5] that to realize equal strength of a body, it is necessary and sufficient to satisfy the equality $B_2 S_2^{(q+1)/2} = C(t)$, which it is expedient to call the optimality condition, at each point at any time $0 < t \leq t_{**}$. In particular, C is independent of the time under stationary external loads and temperature, i.e., is a constant $C = [(\alpha+1)(m+1)t_{**}]^{-1}$; g, α, m are material characteristics, S_2 is the second invariant of the stress tensor deviator, $S_2 = (1/2)s_{ij}s_{ij}$ and the s_{ij} are the stress tensor deviator components. We consider the experimentally established temperature dependence of the coefficient B_2 to have the form [2]: $B_2 = B_0 \exp(c\Theta)$, where B_0, c are material constants, and Θ is the temperature which is a function of the coordinates of the body points. Taking this equality into account, the optimality condition for a body loaded by stationary external loads is written in the form

$$S_2^{(g+1)/2} \exp(c\Theta) = CB_0^{-1}. \tag{1}$$

The system of equations for an optimal body in longevity, that will describe all three stages of creep and take simultaneously into account the damageability of the material, is simplified considerably and takes the form [5]:

$$\eta_{ij} = kS_2^{\lambda} s_{ij}, \quad \lambda = (n-g-2)/2, \ i, j = 1, 2, 3, \quad \omega = (1-\mu)^{1/(\alpha+1)}, \tag{2}$$

where the time functions are

$$\mu = (1 - t/t_{**})^{1/(m+1)}; \quad k = k_1 \left[2(\alpha+1)(m+1)t_{**}\mu^m (1-\mu)^{\alpha/(\alpha+1)}\right]^{-1};$$

and k_1, m, n are material characteristics. Hence the equilibrium equations, the creep strain rate continuity equations, and the appropriate boundary conditions should be satisfied at each point of the body.

Let us consider the solution of the plane axisymmetric problem to determine the external loads and temperature for a body of given geometry that is equally strong during creep. For instance, we consider a thin-walled cylindrical tube in a plane axisymmetric temperature field

$$\Theta(r) = C' + \Theta_* \ln r/a. \tag{3}$$

Here a and r are the inner and running radii of the tube. Let us note that (3) is the solution of the heat-conduction equation for a cylindrical tube under the assumption of no heat transfer at the endfaces for a given convective heat transfer at its inner and outer cylindrical surfaces, or for given temperatures on these surfaces [6]. Under the assumption of this latter, we obtain

$$C' = \Theta(a), \ \Theta_* = [\Theta(b) - \Theta(a)]/\ln\beta, \ \beta = b/a$$

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(b is the outer radius of the tube). Substituting (3) into the optimality condition (1), we obtain the stress intensity distribution law over the tube radius:

$$S_2^{1/2} = A\,(a/r)^{\nu}, \tag{4}$$

where $A = [(\alpha + 1)(m + 1)\, t_{**} B_0 \exp(c\Theta(a))]^{-1/(g+1)}$; $\nu = c\Theta_*/(g + 1)$. Variating the pressure drop Θ_* over the tube radius, and the external loads which are a combination of inner and outer pressures, compliance with condition (4) can be achieved. It is hence evident that the boundary conditions in the temperature and in the external loads cannot be arbitrary. They must be found or appropriate constraints must be indicated. In addition to condition (4), the stress tensor components σ_r, σ_φ should satisfy the equilibrium equation [6]

$$d\sigma_r/dr + (\sigma_r - \sigma_\varphi)/r = 0,$$

while the creep strain rate tensor components η_r, η_φ should satisfy the continuity equation [6]

$$d\eta_\varphi/dr + (\eta_\varphi - \eta_r)/r = 0. \tag{5}$$

Let us introduce the stress function F(r) which satisfies the equilibrium equation identically, where [6]

$$\sigma_r = (1/r)(dF/dr),\ \sigma_\varphi = d^2F/dr^2. \tag{6}$$

As an illustration, let us examine the plane strain case. Taking this into account, and substituting (6) and (2) into the continuity equation (5) with (4) taken into account, we obtain a homogeneous Euler equation in the stress function

$$(n - g - 1)\, d^3F/dr^3 - ((n - g - 3)/r)\, d^2F/dr^2 + ((n - g - 3)/r^2)\, dF/dr = 0.$$

Its general solution has the form

$$F(r) = C_1 + C_2 r^2 + C_3 r^{\nu_1},\quad \nu_1 = 2(n - g - 2)/(n - g - 1). \tag{7}$$

It is possible to set $C_1 = 0$ since C_1 does not influence the stress distribution. The stress function (7) should satisfy the relationship (4) obtained from the optimality condition. Taking (6) into account, (4) becomes for the plane strain case

$$d^2F/dr^2 - (1/r)(dF/dr) = 2A(a/r)^{\nu}.$$

Substituting (7) here and comparing left and right sides, we obtain

$$\nu = 2/(n - g - 1),\ C_3 = -2Aa^{\nu}/\nu\nu_1. \tag{8}$$

Therefore, the stress function (7) finally becomes

$$F(r) = C_2 r^2 - 2Aa^{\nu} r^{\nu_1}/\nu\nu_1,$$

and the stress components (6) here equal

$$\sigma_r = 2C_2 - (2A/\nu)(a/r)^{\nu},\ \sigma_\varphi = 2C_2 - (2A(1 - \nu)/\nu)(a/r)^{\nu}. \tag{9}$$

It is seen from a comparison of the first equation (8) and the relationship $\nu = c\Theta_*/(g+1)$ that the pressure drop over the tube radius cannot be arbitrary. It is determined in terms of the material characteristics and the geometric size of the tube, i.e.,

$$\Theta(b) - \Theta(a) = [2(g + 1)/c(n - g - 1)]\ln\beta. \tag{10}$$

It follows from (9) that the surface loads can also not be arbitrary. They must be selected in such a manner as to equilibrate the radial stresses, say, at the inner and outer surfaces of the tube:

$$\sigma_r(a) = 2C_2 - 2A/\nu,\ \sigma_r(b) = 2C_2 - 2A/\nu\beta^{\nu}.$$

This can be achieved by loading the tube with the inner pressure p_1, the outer pressure p_2 or their combination. In the latter case, we have

$$p_1 - p_2 = s_*(\beta^{\nu} - 1),\ \text{где}\ s_* = 2A/\nu\beta^{\nu}. \tag{11}$$

The integration constant C_2 is obtained equal to $2C_2 = s_* - p_2$, while the stress components finally take the form

$$\sigma_r = -p_2 + s_*[1 - (b/r)^{\nu}],\ \sigma_\varphi = -p_2 + s_*[1 - (1 - \nu)(b/r)^{\nu}]. \tag{12}$$

The strain components ε_r, ε_φ are determined in the form

$$\varepsilon_\varphi = -\varepsilon_r = \varepsilon_*(r)\,\omega(t),$$

after (2) has been integrated with respect to the time, and (4) and (12) have been taken into account, where $\varepsilon_* = 0.5k_1 A^{2/\nu} (a/r)^2$ is the distribution of the strains ε_r, ε_φ over the tube radius at the time of its rupture. It is seen that the stress state (12) in an equal-strength tube during creep is steady, while the strain state is the product of a function of the coordinates by a function of the time. To realized equal strength of a thick-walled tube during creep, it is necessary to give the geometric size of the tube β from the exploitation conditions, the value of the temperature on its inner surface, and the time to rupture t_{**}. By knowing the characteristics of the material from which the tube will be fabricated, we find A and s_* from (4) and (11). From (10) we determine the temperature drop and we thereby find the temperature on the outer surface of the tube. From the first equation in (11) we calculate the pressure drop given first by the inner or outer pressure.

It is interesting to analyze the case when the tube is heated uniformly, i.e., $\Theta_* = 0$. Here $\nu = 0$, and as follows from (4), the stress intensity is independent of the radius and equals $S_2^{1/2} = A$. To be graphic, we set $p_2 = 0$ later. Passing to the limit as $\nu \to 0$ in (11), we find

$$p_1 = 2A\ln\beta.$$

It is seen that this relationship is the analog of the known formula

$$p_* = 2\tau_s \ln\beta,$$

which is used extensively in strength computations of cylindrical tubes and vessels under plastic deformation conditions. Here τ_s is the yield point of the material under pure shear, and A degenerates in the limit into the creep strength of the material defined for a fixed temperature on the basis of t_{**} hours.

It follows from (12) for $p_2 = 0$ and $\nu \to 0$ that the stress distribution corresponds to an ideally plastic state with the sole difference that the quantity p_* in the latter is replaced by p_1, i.e.

$$\sigma_r = -(p_1/\ln\beta)\ln(b/r), \quad \sigma_\varphi = (p_1/\ln\beta)[1 - \ln(b/r)].$$

The case of the plane state of stress can be considered analogously. The method of determining the external temperature–force fields is analogous to that elucidated above. In both cases the boundary conditions in the temperature and the load are not arbitrary. Therefore, they can be difficult to realize technically. In connection with this, the method elucidated to determine the external loads and temperature in order to be able to realize equal strength for specific structure elements during creep can be recommended as additional in the first stages of analysis and design of items. The solution of this problem is very difficult in the most general case.

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